Journal of Sound and Vibration (2001) **241**(5), 925–933
doi:10.1006/jsvi.2000.3320, available online at http://www.idealibrary.com on IDEAL®

OBSERVATIONS ON THE APPEARANCE OF DISPERSIVE PHENOMENA IN DAMAGED STRUCTURES

D. ZONTA

Department of Structural Engineering, University of California San Diego, La Jolla, CA 92093 0085, *U.S.A. E-mail*: *zonta@caronte.dic.unipd.it*

AND

C. MODENA

Dipartimento di Costruzioni e Trasporti, University of Padova, Via Marzolo 9, 35131 *Padova, Italy. E-mail*: *modena@caronte.dic.unipd.it*

(*Received* 14 *August* 2000)

1. INTRODUCTION

From a general standpoint, structural damage can be interpreted as a local variation in mechanical properties and thereby, the global dynamic response is altered owing to damage. In recent years, many proposals have been formulated to verify the integrity of a structure by means of vibrational measures. The most common approach by far consists of modelling damage as a local variation in stiffness, such that in a damaged structure variations in frequency and mode shapes are expected to be found.

The contribution of Adam *et al.* [1] was the first of numerous publications related to the frequency approach. See among others, Salawu's review [2]. The shape-based approach initially relied on the direct comparison of shapes in terms of synthetic parameters such as MAC [3] and COMAC [4]. More recently, the use of mode shapes expressed in terms of curvature [5] or strain [6] has shown advantages. Not so commonly, the detection of structural damage has been proposed on the basis of damping variations, this approach is also present in reference [1], or by identifying non-linearities [7].

Therefore, it would appear that damage detection is essentially based on a comparison of standard modal parameters (shapes, frequencies, and more rarely damping). Conversely, up to now, several other phenomena, which also clearly identify the presence of damage, have been neglected.

1.1. SOME EXPERIMENTAL EVIDENCE

The authors have recently carried out a series of experiments to verify the possibility of using vibrational measures to check the integral state of prefabricated prestressed reinforced concrete (PRC) elements. This paper mainly presents some qualitative aspects of the results. Details of these tests can be found in reference [8].

The free response of one PRC element (i.e., a hollow panel simply supported at its ends) subjected to test is illustrated in Figure 1(a). The response was measured at middle-span level. A load was then applied to the same panel causing the opening of a transverse crack.

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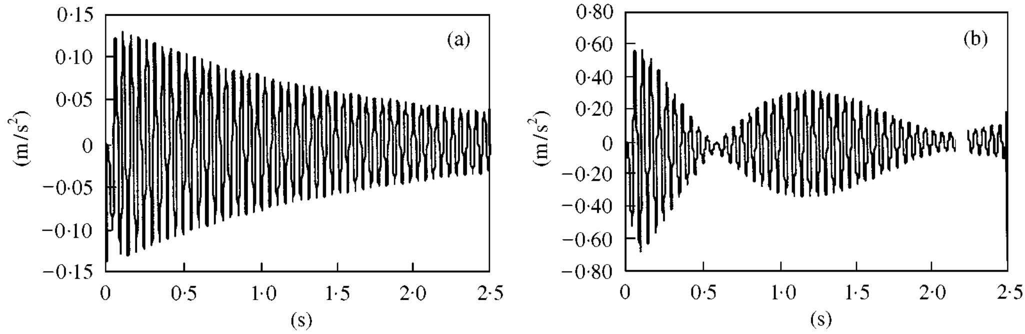

Figure 1. Free response of a PRC specimen: (a) undamaged; (b) cracked.

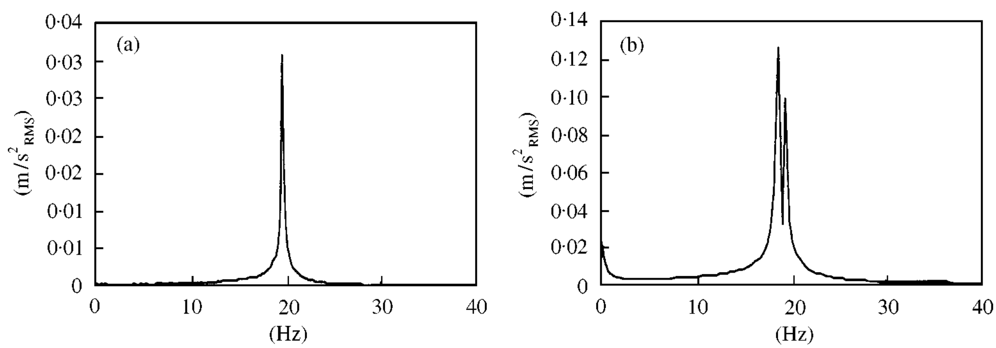

Figure 2. Free response FFT of a PRC specimen: (a) undamaged; (b) cracked.

The free response of the same panel after damage, measured in the same conditions, is shown in Figure 1(b), while Figures 2(a) and 2(b) highlight the Fourier transform of the same signals. Some qualitative remarks on the behavior of the structure before and after damage can be made. The variation of the first frequency can be observed in situation (b), but it is only significant when compared with situation (a). Damping appears to be more sensitive to damage than frequencies. Indeed, it has been demonstrated that in PRC structures the opening of a crack is characterized more clearly by the triggering of non-viscous dissipative mechanisms [8].

Instead, the most evident qualitative difference is the presence of the beat in the damaged structure and its absence in the free signal of the undamaged structure. It consists of a frequency splitting of the resonance peak, in the frequency domain. The appearance of two peaks in the place of one could lead one to think that in an undamaged structure the two frequencies are coupled, and that the damage may have caused a loss of symmetry which made them distinct. A phenomenon of this type has already been studied in the case of prefabricated cylindrical pipes [9].

In this particular case, however, the modal extraction shows that the first peak of the structure is associated with the fundamental bending mode, which is a non-coupled frequency. Moreover, the modal components associated with each of the peaks are substantially the same in the free response of the damaged panel. It appears thereby that the two peaks are related to the same vibration made while the peak splitting seems to be connected to the free response only. Indeed, it appears in the FRF obtained from the shock test but not in those obtained by means of the stepped sinusoidal test. This confirms that the peak splitting is not related to a simple frequency coupling.

The aforementioned phenomenon is not peculiar to a particular material or structural system. Various experiences have shown that frequency splitting is typical of a cracked structure in RC, PRC, or masonry while it is absent in undamaged structures.

1.2. PROPOSITIONS

With this contribution, the authors demonstrate that it is possible to explain the frequency splitting phenomenon through a linear model. In detail: (1) it is sufficient to admit an imaginary damping for an s.d.o.f. system; (2) it suffices to admit that the medium or a localized portion of it is dispersive in the strict sense for a continuous system. Thereby, a dispersion relation

$$\omega^2 + 2\delta v_0 \alpha\omega - v_0^2\alpha^2 = 0 \tag{1}$$

must hold. Due to observation (2), and other reasons illustrated later on, hereinafter the imaginary damping mentioned in observation (1) is referred to as *dispersion.*

2. A GENERALIZED FORM OF THE S.D.O.F. OSCILLATOR

In its most general form, the equation of motion of an unforced s.d.o.f. oscillator is

$$m\ddot{x} + (c + \mathrm{i}d)\dot{x} + (k + \mathrm{i}h)x = 0, \tag{2}$$

where m, c and k are the mass, damping and oscillator stiffness. The coefficient h is well known in the literature, and it is adopted to describe structural or hysteretic damping of the system (see, for example reference [10]). The coefficient d is introduced here and is defined as system *dispersion.* Equation (2) generalizes the most standard form of the oscillator equation, where the terms considering velocity and displacement are real. By normalizing each term through the mass and assuming

$$\frac{k}{m} = \omega_n^2, \quad \frac{h}{m} = \eta\omega_n^2, \quad \frac{c}{m} = 2\xi\omega_n, \quad \frac{d}{m} = 2\delta\omega_n, \tag{3}$$

a normalized equation of motion is obtained:

$$\ddot{x} + 2\omega_n(\xi + \mathrm{i}\delta)\dot{x} + \omega_n^2(1 + \mathrm{i}\eta)x = 0. \tag{4}$$

The *dispersion rate* is defined as the nondimensional value δ, in analogy with the definition of damping rate attributed to ξ. The equation of motion has solutions of the type

$$x(t) = A\mathrm{e}^{s_1 t} + B\mathrm{e}^{s_2 t}, \tag{5}$$

where s_1 and s_2 satisfy the characteristic equation

$$s^2 + 2\omega_n(\xi + \mathrm{i}\delta)s + \omega_n^2(1 + \mathrm{i}\eta) = 0. \tag{6}$$

Evidently, they also satisfy the equation

$$s^2 - (s_1 + s_2)s + s_1 s_2 = 0. \tag{7}$$

Thereby, from a comparison between equations (6) and (7) it can be deduced that

$$Re(s_1 + s_2) = -2\xi\omega_n, \quad Im(s_1 + s_2) = -2\delta\omega_n. \tag{8}$$

On the other hand, the following relation holds:

$$|Im(s_1 + s_2)| = ||Im\, s_1| - |Im\, s_2|| = |q_1 - q_2| = \Delta q, \tag{9}$$

where Δq represents the frequency splitting. From a comparison of equations (8) and (9), it can be deduced that

$$\Delta q = 2\delta\omega_n, \tag{10}$$

that is, the frequency splitting appears if, and only if, the dispersive term is not equal to zero (Figure 3).

2.1. FREE RESPONSE OF A DISPERSIVE OSCILLATOR

Hereinafter, the free response of an undamped, non-hysteretic oscillator with the dispersive term is analyzed. Under these assumptions, the equations of motion reads as

$$m\ddot{x} + \mathrm{i}d\dot{x} + kx = 0, \tag{11}$$

which in normalized form becomes

$$\ddot{x} + \mathrm{i}2\omega_n\delta\dot{x} + \omega_n^2 x = 0. \tag{12}$$

The solutions of the characteristic equation

$$s^2 + \mathrm{i}2\omega_n\delta s + \omega_n^2 = 0 \tag{13}$$

lead to the solutions

$$s_1 = +\mathrm{i}\omega_n(\sqrt{1-\delta^2} - \delta) = +\mathrm{i}q_1, \quad s_2 = -\mathrm{i}\omega_n(\sqrt{1-\delta^2} + \delta) = -\mathrm{i}q_2. \tag{14}$$

Thereby, equation (11) admits two imaginary oscillating solutions, with pulsations q, one greater and the other smaller than the natural frequency according to the plot of Figure 4. The time history has a form of the type

$$x(t) = C_1 \mathrm{e}^{\mathrm{i}q_1 t} + C_2 \mathrm{e}^{-\mathrm{i}q_2 t}, \tag{15}$$

with C_1 and C_2 being complex coefficients which can be calculated using the initial conditions. Equation (14) describes the beat phenomenon very clearly and also underlines that the d term is conservative. It would therefore be misleading to talk of imaginary damping. In the same way, the hysteretic term h, which is the *imaginary stiffness*, is dissipative indeed.

2.2. FORCED RESPONSE

Hereinafter, the forced response of a dispersive oscillator is analyzed. It is therefore necessary to find a particular solution of the equilibrium equation

$$m\ddot{x} + \mathrm{i}d\dot{x} + kx = F_0 \mathrm{e}^{\mathrm{i}\omega t}. \tag{16}$$

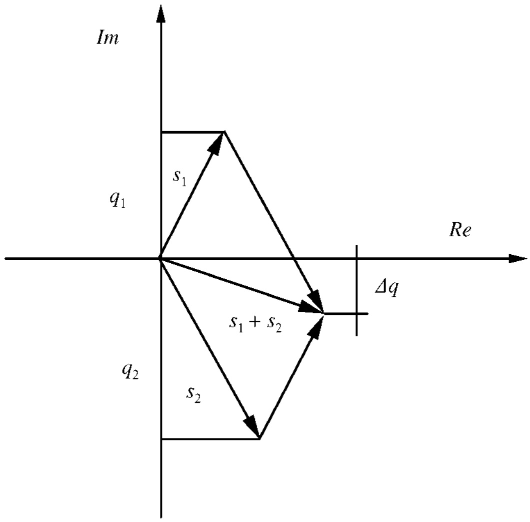

Figure 3. Representation of the frequency splitting in the complex plane.

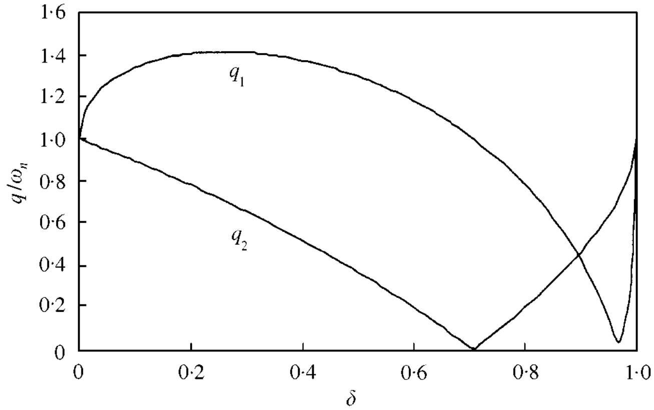

Figure 4. Natural frequency variation of the dispersive oscillator versus the dispersion rate.

Solutions of the type

$$x(t) = x_0 e^{i\omega t} \tag{17}$$

are sought, which when substituted into equation (16) provide

$$(-\omega^2 m - \omega d + k)x_0 = F_0, \tag{18}$$

from which the expressions both of the dynamic stiffness

$$R(\omega) = (-\omega^2 m - \omega d + k) \tag{19}$$

and of the receptance

$$\alpha(\omega) = \frac{F_0}{x_0} = \frac{1}{-\omega^2 m - \omega d + k} = \frac{1}{k}\frac{1}{1 - 2\delta(\omega/\omega_n) - (\omega/\omega_n)^2} \tag{20}$$

can be found.

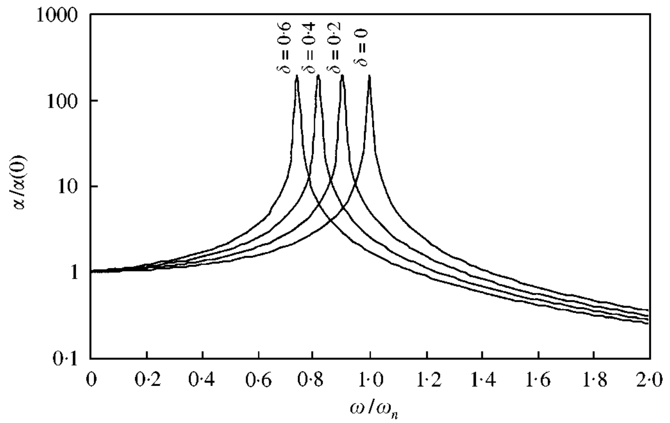

Figure 5. FRF of a dispersive oscillator for different values of the dispersion rate.

The receptance function is real and produces a non-finite value corresponding to the resonance. The relevant graph is shown in Figure 5 together with the frequency response of the simple oscillator. The resonance is obtained by assuming that the dynamic stiffness expressed by equation (20) is null. Thus, obtaining two solutions

$$\bar{\omega}_{1,2} = \omega_n(-\delta \pm \sqrt{\delta^2 + 1}). \tag{21}$$

However, only one solution, the positive one,

$$\bar{\omega} = \omega_n(\sqrt{\delta^2 + 1} - \delta), \tag{22}$$

is endowed with a physical meaning. Thereby, only one resonance peak in the FRF exists whose value is lower than the one of the simple oscillator.

2.3. OBSERVATION

From equation (19), it is easy to prove that the presence of the dispersive term is equivalent to admitting that the oscillator embodies a variable stiffness proportional to the frequency:

$$k'(\omega) = k - \omega d. \tag{23}$$

3. AN INTERPRETIVE MODEL FOR CONTINUOUS SYSTEMS

A one-dimensional continuous system is considered. Physically, it can represent a bar composed of a non-dissipative homogeneous elastic material. The equilibrium equation, which describes the propagation of perturbation, u, along the medium, is the classical wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}, \tag{24}$$

where v is the propagation velocity, which in the case of pressure waves reads as

$$v = \sqrt{E/\rho}. \tag{25}$$

In the literature [11], the nomenclature *dispersive system* has two different meanings. In a broad sense, a dispersive system is any system which allows solutions of the form

$$u = A\cos(\alpha x - \omega t), \tag{26}$$

where ω is the frequency and α the wave number. Equation (24) admits solutions of this type. An expression that relates the frequency to the variation of the wave number is called a *dispersion relation*. With regard to equation (24), a linear relationship holds:

$$v = \omega/\alpha. \tag{27}$$

In a strict sense, a dispersive system is one that admits a non-linear dispersion relation or, in other words, a system in which the propagation velocity varies with the wave frequency. As a result, a system that is described through the classical wave equation is not dispersive in a strict sense.

Hereinafter, a formulation that reproduces the frequency splitting phenomenon in the case of a continuous system is found. In particular, a wave equation which admits two different frequencies for the same wave number is sought. It is evident that dispersion relation (27) associated with the classical wave equation, cannot describe that phenomenon, as a single frequency is associated with one wave number. This means that the system being sought is dispersive in a strict sense. Equation (23) suggests studying a mechanical system where the modulus of elasticity varies linearly with the frequency, according to the expression

$$E(\omega) = E_0 + \gamma\omega, \tag{28}$$

where γ is a constant. Keeping in mind equation (25), it is possible to write an analogous equation for the phase velocity

$$v^2 = v_0^2 - \gamma\omega \tag{29}$$

from which the following dispersion relation is obtained:

$$\frac{\omega^2}{\alpha^2} + \gamma\omega + v_0^2 = 0. \tag{30}$$

Equation (30) is analogous to the characteristic equation (6) when one assumes that

$$\delta = \frac{\gamma}{2v_0}. \tag{31}$$

By substituting the relationship (29) within (24), one obtains

$$\frac{\partial^2 u}{\partial t^2} = v^2\frac{\partial^2 u}{\partial x^2} - 2\delta v^2\frac{\partial^2 u}{\partial x^2}. \tag{32}$$

For the sinusoidal wave, it also holds that

$$\omega v^2 \frac{\partial^2 u}{\partial x^2} = v \frac{\partial^2 u}{\partial x \partial t} \tag{33}$$

and the wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} - 2\delta v \frac{\partial^2 u}{\partial x \partial t}, \tag{34}$$

which describes the frequency splitting phenomenon in a continuous system, is obtained. The findings obtained above suggest the possibility of modelling localized damage as part of a continuum, in which the wave propagation follows equation (34). In a similar manner, the common damage simulation allows the modelling of a part of a continuum with reduced stiffness.

4. CONCLUSIONS

Some experimental evidence has shown that cracking in reinforced concrete or masonry structures is accompanied by a dispersive phenomenon, which has been named as frequency splitting. As this phenomenon is easily recognizable and appears with regularity, it could be adopted in damage detection.

It has been demonstrated that the frequency splitting phenomenon cannot be described with classical modal analysis tools. However, it can be reproduced by a linear model which embodies a skew-symmetric damping operator. For an SDOF system, this corresponds to the presence of an imaginary damping while for continuous systems it is due to the presence of a mixed second order differential operator in the wave equation. In this sense, the proposed model presents formal analogies to the equations of motion in gyroscopic systems [12]. Nonetheless, the nature of this phenomenon is clearly different in a damaged structure and not easily detected microscopically.

Finally, it should be emphasized that the presence of damage exhibits itself not only in the variation of standard modal parameters of the structural dynamic response. Additional phenomena appear in the form of non-linearities, hystereses, non-classical dissipative mechanisms, dispersion, etc., which deserve further studies.

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